WOTA@Waterfront 2022, Geelong, Deakin University

Organisers:

Dr Reinier Diaz Millan	Deakin University
Dr Julien Ugon	Deakin University
Dr Vinesha Peiris	Deakin University
Dr Nadia Sukhorukova	Swinburne University of Technology

Sponsors:

Mathematics of Computation and Optimisation (MoCaO) Special Interest Group of the Australian Mathematical Society (AustMS)

Deakin University

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Abstracts

TBA

Adil Baghirov
Federation University Australia
Email: a.bagirov@federation.edu.au

TBA

A proximal subgradient algorithm with extrapolation for structured nonconvex nonsmooth problems

Minh Dao
Federation University Australia
Email: m.dao@federation.edu.au

In this work, we consider a class of structured nonconvex nonsmooth optimization problems, in which the objective function is formed by the sum of a possibly nonsmooth nonconvex function and a differentiable function with Lipschitz continuous gradient, subtracted by a weakly convex function. This type of structured problems has many practical applications such as signal recovery, clustering, and compressed sensing. We develop a flexible extrapolated proximal subgradient algorithm for solving these problems with guaranteed subsequential convergence to a stationary point. The global convergence of the whole sequence generated by our algorithm is also established under the Kurdyka–Lojasiewicz property. To illustrate the promising numerical performance of the proposed algorithm, we conduct numerical experiments on a compressed sensing problem with a nonconvex regularization and an optimal power flow problem with distributed energy resources.

Approximate algorithms for two-sets convex feasibility problems

Reinier Díaz Millán
Deakin University
Email: r.diazmillan@deakin.edu.au

In this paper, we propose new algorithms combining the Alternating Projection and Douglas-Rachford (DR) algorithms and the Frank-Wolfe algorithm, also known as the conditional gradient (CondG) method, for solving the classic convex feasibility problem. Within the algorithm, which will be named *Approximate Douglas-Rachford (ApDR) algorithm*, the CondG method is used as a subroutine to compute feasible inexact projections on the sets under consideration, and the ApDR iteration is defined based on the DR iteration. The ApDR

algorithm generates two sequences, the main sequence, based on the DR iteration, and its corresponding shadow sequence. When the intersection of the feasible sets is nonempty, the main sequence converges to a fixed point of the usual DR operator, and the shadow sequence converges to the solution set. We provide some numerical experiments to illustrate the behaviour of the sequences produced by the proposed algorithm.

The p-Lagrangian relaxation for nonconvex MIQCQP problems with complicating constraints

Andrew Eberhard
RMIT
Email: andy.eberhard@rmit.edu.au

We presents a novel technique to solve nonconvex mixed-integer quadratically constrained quadratic programming (MIQCQP) with separable structures, such as those arising in deterministic equivalent representations of two-stage stochastic programming problems.

Joint work with T. Andrade, N. Belyak, S. Hamacher, and F. Oliveira.

Radius theorems revisited

Alex Kruger
Federation University Australia and RMIT
Email: akrugeremail@gmail.com

The topic of radius of "good behaviour" quantifying the "distance" of a given well-posed problem to the set of ill-posed problems of the same kind was explicitly initiated by Dontchev et al. in [1,2]. The authors studied the "good behaviour" of generalised equations characterised by regularity properties of set-valued mappings and established in finite dimensions exact formulas for the radii for the three fundamental properties: metric regularity, strong metric regularity and strong metric subregularity with respect to calm, Lipschitz continuous and linear perturbations in terms of the modulus of the respective property. In infinite dimensions, they obtained certain lower estimates for the radii. They also showed that for the property of (not strong) metric subregularity the analogues of the mentioned formulas and estimates fail. The radius of metric subregularity in finite dimensions was studied in [3], where new primal-dual derivatives of set-valued mappings and several moduli of subregularity were introduced and employed for characterising the radius with respect to the same and certain new classes of perturbations. The infinite dimensional case is studied in [4,5]. More moduli of subregularity and more classes of perturbations are considered. The radius theorems for metric regularity and strong metric subregularity from [2] are improved. In the talk, I am going to summarise the developments in [1-5].

Joint work with Asen Dontchev, Helmut Gfrerer and Jiří Outrata In memory of Asen Dontchev.

References

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4. Gfrerer H., Kruger A.Y.: Radius theorems for subregularity in infinite dimensions. arXiv 2206.10347 (2022).

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Edge connectivity of simplicial polytopes

Guillermo Pineda Villavicencio

Deakin University

Email: guillermo.pineda@deakin.edu.au

We show that the graph of a simplicial polytope of dimension $d \geq 3$ has no nontrivial minimum edge cut with fewer than $d(d+1)/2$ edges, hence the graph is $\min\{\delta, d(d+1)/2\}$-edge-connected where δ denotes the minimum degree. When $d = 3$, this implies that every minimum edge cut in a plane triangulation is trivial. When $d \geq 4$, we construct a simplicial d-polytope whose graph has a nontrivial minimum edge cut of cardinality $d(d+1)/2$, proving that the aforementioned result is best possible.

Approximation of functions by neural networks and rational functions

Vinesha Peiris

Deakin University

Email: v.peiris@deakin.edu.au

Rational functions and rational approximation techniques are very efficient and powerful tools that have been widely used for function approximation. Lately, deep learning has gained attention due to its recent success in many different sectors. Deep learning techniques are based on neural networks, which contain a certain number of layers to perform several mathematical transformations on the input. In this talk, we discuss 6 distinct approaches

(4 different neural networks, Differential Correction method and AAA algorithm) that one can use to approximate a given function. In order to determine which technique works best for function approximation, we compare the computational time and the objective function value for each method.

Hyperbolicity cones are amenable

James Suanderson

Monash University

Email: james.saunderson@monash.edu

Hyperbolicity cones are a family of algebraically structured closed convex cones that include all spectrahedral cones (linear sections of positive semidefinite cones) as special cases. The Lax conjecture asserts that all hyperbolicity cones are spectrahedral. If this were true, then various geometric properties of spectrahedral cones should also hold for hyperbolicity cones. This talk will discuss recent results in this direction, with a focus on amenability, a notion of facial exposedness for convex cones that is stronger than being facially dual complete (or "nice") which is, in turn, stronger than merely being facially exposed.

Joint work with Bruno F Lourenço and Vera Roshchina.

The deep mathematical nature of deep learning

Nadia Sukhorukova

Swinburne University of Technology

Email: nsukhorukova@swin.edu.au

Deep learning is one of the key tools in the modern area of Artificial Intelligence. This recognition is well-deserved due to many practical applications, where deep learning demonstrated its efficiency, including data analysis and signal and image processing and many others.

The origin of deep learning is mathematical in its nature. Essentially, the objective of deep learning is to solve an approximation problem: optimise the weights (parameters) of the network. These weights can be considered as decision variables of certain optimisation problems, whose objective functions represent inaccuracy. Therefore, it is natural to approach this problem can be treated using modern optimisation tools and this is one of the modern research directions in the field of deep learning.

In this talk we provide a short review of the mathematical developments in the field of deep learning and identify a number of interesting optimisation problems.

Optimization Approach Towards Improving Separability of Clusters

Sona Taheri
RMIT
Email: sona.taheri@rmit.edu.au

The objective functions in optimization models of the sum-of-squares clustering problem reflect intra-cluster similarity and inter-cluster dissimilarities and in general, optimal values of these functions can be considered as appropriate measures for compactness of clusters. However, the use of the objective function alone may not lead to the finding of separable clusters. To address this shortcoming in existing models for clustering, we introduce a new optimization model where the objective function is represented as a sum of two terms reflecting the compactness and separability of clusters. Based on this model we develop a two-phase incremental clustering algorithm. In the first phase, the clustering function is minimized to find compact clusters and in the second phase, a new model is applied to improve the separability of clusters. Starting cluster centers are generated using data points located on the boundary of clusters. The Davies-Bouldin cluster validity index is applied as an additional measure to compare the compactness of clusters and silhouette coefficients are used to estimate the separability of clusters. The performance of the developed algorithm is evaluated using some synthetic and real-world data sets.

Forward-backward splitting without cocoercivity

Matthew Tam
University of Melbourne
Email: matthew.tam@unimelb.edu.au

The forward-backward splitting algorithm is a method for finding a zero in the sum of two monotone operators when one assumed single-valued and cocoercive. Unfortunately, the latter property, which is equivalent to strong monotonicity of the inverse, is too strong to hold in many monotone inclusions of interest. In this talk, I will discuss a recently discovered modification of the forward-backward splitting algorithm which converge without requiring the cocoercivity assumption. Applications in training generative adversarial networks (GANs) will be discussed.

This talk is based on joint work with Yura Malitsky (Linköping).

Relaxation convergences for robust polynomial programs with applications.

Chuong Thai Doan
RMIT
Email: chuong.thai.doan@rmit.edu.au

In this talk, we consider a bilevel polynomial bilevel problem, where the constraint functions of both the upper-level and lower-level programs involve uncertain parameters. We employ the deterministic robust optimization approach to examine the bilevel polynomial optimization problem under data uncertainty by providing lower bound approximations and convergences of sum-of-squares relaxations for the robust bilevel polynomial optimization problem. More precisely, we show that the global optimal values of sum-of-squares relaxation problems are lower bounds of the global optimal value of the robust bilevel polynomial problem and they converge to this global optimal value when the degrees of sum-of-squares polynomials in the relaxation problems tend to infinity. Moreover, an application to electric vehicle charging scheduling problem with renewable energy sources demonstrates that using the proposed sum-of-squares relaxation schemes we obtain more stable optimal values than applying a direct solution approach as the sum-ofsquares relaxations are capable of solving these models involving data uncertainties in dynamic charging price and weather conditions.

Investigating the variational properties of the abstract subdifferential

Julien Ugon
Deakin University
Email: julien.ugon@deakin.edu.au

The classical notions of subgradient and subdifferential play an essential role in convex analysis, due to its several remarkable properties such as it being a maximal monotone operator, and the fact that it largely enjoys a full calculus. For that reason many generalisations to nonconvex functions have been proposed. In this talk we review the abstract subdifferential and present some preliminary observations on which properties of the convex subdifferential generalise well, and which ones require more care.

Polytopes with low excess degree

David Yost
Federation University Australia
Email: d.yost@federation.edu.au

Any vertex of a d-dimensional polytope has degree at least d. We define the excess degree of a vertex as its degree minus the dimension, and the excess degree of a polytope as the sum of the excess degrees of its vertices. The excess degree of a polytope is easily checked to equal $2e - dv$, where v and e are the numbers of vertices and edges. We show that having low excess degree (no more than $2d - 6$) imposes some strong restrictions on the structure of a polytope.

Joint work with Guillermo Pineda-Villavicencio and Jie Wang.